LINEAR ESTIMATORS OF THE EXPORT EMPLOYMENT MULTIPLIER

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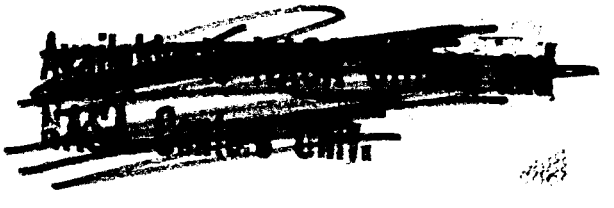

Linear Estimators of the Export Employment Multiplier

by Edwin F. Terry*

The idea in multiplier models is to regard certain types of spending as a determinant of other spending. The ratio of the total spending derived from the change in determinant spending is the income multiplier concept. The employment multiplier is the ratio of the employment changes accompanying the total and determinant spending changes.

This paper is a clarification of the assumptions implicit in estimators frequently used in making estimates of the aggregate export employment multiplier for urban economies. The terms export and import are used to include trade of a subnational region with the rest of the nation as well as with foreign countries. The principal innovator of this employment multiplier in concept and practice was Homer Hoyt.1/ Much of the literature concerning the export employment multiplier uses the word basic as a rough synonym for export.

Investment Multiplier

The first thing that will be established in developing an export employment multiplier model is its monetary mechanism, the investment multiplier. To derive the investment to be used in conjunction with the investment multiplier, gross regional product produced ($\bar{GRP}_p$) will be broken down in a simplified fashion twice, once by final demand categories and once by income payments. The derivation of this investment will be seen to be invariant to the use of gross regional product produced or received.

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1. Hoyt, Homer. "Homer Hoyt on Development of Economic Base Concept." Land Economics, Vol. 41, May, 1954, pp. 173-198.

By final demand categories, GRPp is composed of consumption (C) plus investment (I') plus regional government (G'). Investment is composed of

+ gross private capital formation	(GPCF')
+ inventory additions	(inv add)
- inventory depletions	(inv dep)
+ exports	(E')
- imports	(M)

It is the omission of net in-area factor owner income flow in investment (I') which causes these components to sum to the production, rather than the receipt of gross regional product. Gross private capital formation (GPCF') relates to all real capital formation by the private sector located within the region. Exports (E') here consist of all private sector sales to nonresidents and business and government establishments not located in the region. Imports (M) include all imports by residents and regional establishments. Regional government is all nontransfer expenditures by local, state and federal government establishments located in the region. 2/

Gross regional product produced on a simplified income payments basis is equal to the following sum:

	abbreviation
Net personal savings	NPS
+ out-area gifts by residents	out Tf
+ consumption	C
= Disposable income	
+ personal taxes	Tx_1
= Personal income	
+ all out-area factor owner flow	out Flo
- in-area factor owner flow to residents	in Flo
- in-area gifts to residents	in Tf
- all government transfer payments to residents	govt Tf
+ business social security taxes	Tx_2
+ net corporate saving	NCS
+ corporate profits tax	Tx_3

2. For a discussion of the regional government account, see: Burkhead, Jesse. "Public Finance as an Integral Part of Regional Accounts." In: Elements of Regional Accounts. Werner Hirsch, editor. Johns Hopkins Press. Baltimore, Md. 1964. pp. 60-65.

	abbreviation
= Regional factor income produced	
+ Indirect business tax	Tx_4
= Net regional product	
+ depreciation	Dep
= Gross regional product produced	GRPp

The two breakdowns of GRPp yield the following economic identity, dropping numerical subscripts to denote summation:

$$C + GPCF' + \text{Inv add} - \text{inv dep} + E' - M + G' \equiv NPS + \text{out Tf} + C + Tx + \text{out Flo} - \text{in Flo} - \text{in Tf} - \text{govt Tf} + NCS + Dep.$$

Consumption cancels out. Also, in equilibrium, inventory additions and depletions will not be present. The equilibrium components of the previous identity can be rearranged so that

$$GPCF' + E' + G' + \text{in Flo} + \text{in Tf} + \text{govt Tf} = M + NPS + \text{out Tf} + Tx + \text{out Flo} + NCS + Dep.$$

Multiplier investment (I) will be defined to consist of those components to the left of the previous equality and leakage will consist of the right hand side components.

Regional consumption and leakage are regarded as functions of investment (I) in multiplier analysis. If the consumption function is linear and homogeneous, the product of total investment times the investment multiplier is equal to the sum of that investment plus all consumption by regional residents. The sum of multiplier investment and consumption will be denoted by Y. By comparison with the preceding income account discussion Y, equal to $C + GPCF' + E' + G' + \text{In Flo} + \text{In Tf} + \text{govt Tf}$, is seen to be equal to equilibrium GRPp + In Flo + In Tf + govt TF + Imports.

The economic derivation of the investment multiplier from the consumption function is well known. Briefly, a constant fraction r ($0 < r < 1$) of investment expenditure is assumed to be spent on consumption by all regional recipients. That fraction is referred to as the marginal propensity to consume. Induced regional consumption is then equal to the sum of

the infinite geometric series $I(r + r^2 + r^3 + \ldots) = I(1/(1-r)-1)$. That amount of consumption not induced by investment is indicated by the symbol γ.

$$\text{So } Y = I + C$$
$$= I\,(1/(1-r)) + \gamma$$
$$\text{and } \Delta Y = \Delta I\,(1/(1-r))$$

The quantity $1/(1-r)$ is the investment multiplier. Regardless of the form of the induced consumption function, $\Delta Y/\Delta I$ for the interval $I_0 + \Delta I$ is approximately equal to dY/dI evaluated at I_0 assuming the function to possess a derivative at I_0 and the disturbance term in stochastic functions to be independent of investment. The investment multiplier is usually denoted by K.

The leakage function can be found by solving for I since multiplier investment will be equal to leakage in equilibrium. In the linear example:

$$I = (1-r)\,(Y-\gamma)$$
$$= (1-r)\,Y - (1-r)\,\gamma$$
$$= (1/k)\,Y + Q$$

where $1 - r = 1/K$ is the regional marginal propensity to leak.

Employment Multiplier Models

An employment multiplier corresponding to the preceding investment multiplier will be derived first.

Let N = total employment in a region, a function of Y
N_I = investment employment, a function of multiplier investment (I) with a one to one correspondence

Then $\Delta N = (dN/dY)\ (dY/dI)\ (dI/dN_I)\ \Delta N_I$

so that $\Delta N/\Delta N_I = (dN/dY)\ k\ (dI/dN_I)$

This model of the investment employment multiplier is essentially the same as that given by Keynes for Kahn's investment employment multiplier. 3/

3. Keynes, J. M. The General Theory of Employment, Interest and Money. Harcourt, Brace and Co. New York, N.Y. 1936. p. 116.

The components of multiplier investment and the resulting real activity can be thought of as arising to serve two groups: (1) regional residents and (2) nonregional residents and establishments. That portion of investment allocable to residents will be designated domestic investment (D) and that allocable to the nonresident group exports (E) for purposes of constructing an export employment multiplier model. The employment associated with D and E will be denoted by Nd and Ne, respectively. In effect, this concept of exports and domestic investment converts some of the final demand components of multiplier investment to a derived demand status. The problem is to allocate this intermediate production to the two classes of ultimate final demand, exports and regional residents. After this is done, multiplier exports will consist of that investment not arising in response to regional resident demand or needs. It would then be considered to more adequately fill the role of the sole exogenous element in regional growth it is stated to be by the export employment multiplier than, for example, exports as defined in the income accounts. A methodology for accomplishing this allocation is sketched in the following.

Included in multiplier exports would be all of income account exports (E') with domestic investment including the in-flow, private in-transfer and government transfer components of multiplier investment. An allocation of gross private capital formation and regional government is more involved. The possibility that some regional gross private capital formation as defined in the income accounts arises in anticipation of export sales potential must be allowed. By this thinking, those GPCF' items should be allocated to exports and domestic according to the sales anticipations for the output to be produced by the real investment. It should be noted that the correctness of the sales anticipations would have nothing

to do with the validity of this allocation. In the investment decision such sales anticipations, if they exist, may extend only to the first buyer of the output. If so, only the direct export sales fraction would be anticipated. Some regional buyers also might be direct exporters and/or some of the buyers' regional customers might export, and so on. Hence, the direct and indirect "anticipated" export sales fraction could be larger. Only the direct anticipations will be considered here, however, because of the theoretical and empirical problems involved in quantifying the indirect export sales anticipations portion of GPCF'. For purposes of discussion, the income account gross private capital formation (GPCF') is divided into two parts, an export portion included in E and domestic gross private capital formation, denoted by GPCF, included in D.

An export and domestic empirical allocation of regional government expenditures, with the exception of user-cost-financed activities, would have to be a judgement based on a general knowledge of the role of the agencies. The same indirect export estimation problem exists for this sector as for capital formation. 4/ Regional government spending would be divided into capital and noncapital spending. Public capital spending would be allocated according to the anticipated users and noncapital spending according to the actual users. Examples of regional government allocation are regional military establishment construction or operation which would be export and the construction or operation of post offices which could largely be domestic investment. Domestic regional government expenditures will be denoted by G.

4. A theoretical construct which could accomplish the allocation more rigorously is discussed in: Perloff, H. S. and Leven, C. L. "Towards an Integrated System of Regional Accounts: Stocks, Flows, and the Analysis of the Public Sector." In: Elements of Regional Accounts, op. cit., pp. 197-204.

In the investment employment multiplier model, investment was regarded as a function of one type of employment so that $\Delta I = (dI/dN_i)\Delta N_i$. In the export employment multiplier, investment is regarded as a function of two types of employment so that $\Delta I = (\partial I/\partial N_d)\Delta N_d + (\partial I/\partial N_e)\Delta N_e$. A composite function is encountered in evaluating the partials. Diagrammatically:

$$I \begin{cases} D - N_d \\ E - N_e \end{cases}$$

Therefore, $\partial I/\partial N_d = (\partial I/\partial D)(dD/dN_d) = 1 \cdot (dD/dNd) = dD/dN_d$ and similarly $\partial I/\partial N_e = dE/dN_e$. The export employment multiplier model corresponding to the investment model is .

$$\frac{\Delta N}{\Delta N_e} \simeq \frac{dN}{dY} \cdot \frac{dY}{dI}\left(\frac{\partial I}{\partial Nd} \cdot \frac{\Delta Nd}{\Delta N_e} + \frac{\partial I}{\partial N_e}\right)$$

$$\simeq \frac{dN}{dY}\ k\ \left(\frac{dD}{dNd} \cdot \frac{\Delta Nd}{\Delta Ne} + \frac{dE}{dNe}\right)$$

It will be noted that the export employment multiplier model supposes knowledge of the ratio of the change in domestic investment employment to the change in export employment. The four equations explicitly suggested by the derivatives in this export employment multiplier model are referred to as structural.

Employment Multiplier Estimators

Conditions sufficient for three frequently used estimators of the export employment multiplier to be in agreement with the preceding model will now be developed. The first estimator assumes the ratio of change in total employment to the change in export employment to be equal to their average ratio. 5/ That is, $\Delta N/\Delta N_e = N/N_e$, therefore $N = \beta' N_e$. The following

5. For an example, see: City of Wichita. Patterns for Progress, 1957, an Economic Base Study of the Wichita, Kansas Metropolitan Area. 1957.

five linear homogeneous structural relations would enable this estimator to correspond to the export employment multiplier model just presented. Let:

$Y = \beta_n N$	$D = \beta_d N_d$	$N_d \geq 0$
leakage $= (1/k)\ Y$	$E = \beta_e N_e$	$N_e > 0$
$I = D + E$		$N > 0$

For stability, the income account-employment functions would have to be measured in constant prices.

The export employment multiplier can be derived by starting with the condition necessary for equilibrium Y, namely planned leakage equal to planned investment. From the preceding

$$(1/k)\,Y = I$$

substituting

$$\beta_n N = k\,(D + E)$$

$$N = (k/\beta_n)\ (\beta_d N_d + \beta_e N_e)$$

$$= (k/\beta_n)\ (\beta_d N_d/N_e + \beta_e) N_e$$

denote this by $N = \beta' N_e$

then $\Delta N/\Delta N_e = N/N_e = \beta'$

It can be seen that $\beta' = \frac{dN}{dY}\, k \left(\frac{dD}{dNd}\,\frac{Nd}{Ne} + \frac{dE}{dNe}\right)$

Since N_d/N_e is constrained to be a constant, $\Delta N_d/\Delta N_e$ must always be equal to it. If this is true, and if the structural relations are linear and homogeneous, the estimator will correspond to the model. The model expression for the export employment multiplier will be without any error attributable to the linear nature of an approximation by the use of differentials in this instance. This estimator, referred to as the linear and homogeneous estimator, can be implemented with estimates of export and total employment for one time period.

A second estimator of the export employment multiplier assumes total employment to be linearly regressed on export employment, $N = \delta + \beta'' N_e$. 6/

6. For an example, see: Kansas City Federal Reserve Bank. "The Employment Multiplier in Wichita." Monthly Review, Sept., 1952.

The ratio $\Delta N/\Delta N_e$ is then equal to the linear regression coefficient β''. The following structural relations would be sufficient for this estimator to correspond to the export model. Let:

$$Y = \tau + \beta_n N \qquad 0 \leq m \leq 1$$

$$\text{leakage} = Q + (I/\kappa)\, Y \qquad N_d \geq 0$$

$$) = D + E \qquad N_e > 0$$

$$D = m\,\ell + \beta_d N_d \qquad N > 0$$

$$E = (1-m)\,\ell + \beta_e N_e$$

The export employment multiplier is derived as before by equating planned leakage and investment and substituting $N_d/N_e = \varepsilon$.

Then $N = \frac{-\tau + \kappa(\ell - Q)}{\beta_n} + \frac{\kappa}{\beta_n}(\beta_d \varepsilon + \beta_e) N_e$, denote this by $N = \delta + \beta'' N_e$ so that $\beta_n \Delta N/\Delta N_e = \beta''$.

β'' is seen to correspond to the export employment multiplier model because its functional content is identical to the previous β'. If N_d/N_e is equal to a constant so that $\Delta N_d/\Delta N_e$ is the same constant, and if the structural relations are linear but not necessarily homogeneous, the expression for β'' will be in agreement with the export employment multiplier model. The model will again be without error attributable to its previously referred to linearity. This estimator, termed the linear nonhomogeneous estimator, can be implemented with estimates of export and total regional employment for two time periods.

The linear regression of total employment on export employment could have been derived without any economic argument by resubstituting investment (I) for leakage and writing the inverse of the first two structural relations.

The identical composite function $N = \delta + \beta'' N_e$ can then be constructed from the structural relations by simple substitution. This procedure would

work for the first estimator as well and will be used to derive the third estimator.

If estimates of export and total employment are available for more than two time periods, they will not usually be numerically consistent with the linear estimator previously examined. The addition of a disturbance term to the linear nonhomogeneous estimator and a least squares estimate of the regression coefficient defines the third estimator of the export employment multiplier which will be termed the stochastic linear estimator. 7/ Mathematically, the stochastic regression equation is $N = \delta + \beta N_e + u$.

The addition of a disturbance term to the linear nonhomogeneous estimator indicates that one or more of its structural relations must have a disturbance term. Before these terms are added, the preceding structural relations will be written in different form. The first relation $Y = \tau + \beta''_n N$ is written in inverse form as $N = \ell_1 + \beta_1 Y$. The second relation, $I = Q + (1/k)\ Y$, resubstituting multiplier investment (I) for leakage, is written in inverse form as $Y = \ell_2 + kI$. The export and export employment and domestic investment-domestic employment relations are substituted in the $I = D + E$ identity yielding

$$I = m\ \ell + \beta_d N_d + (1-m)\ \ell + \beta_e N_e$$

$$= \hat{\ell} + \hat{\beta}_d \hat{N}_d + \beta_e N_e$$

This will be rewritten as $I = \ell + \beta_2 (N_d + N_e) = \ell + \beta_2 N_i$ utilizing the equality $N_i = N_d + N_e$. It remains to demonstrate the existence of the constant β_2. This requires that $\beta_d = \beta_e = \beta_2$ or that N_d and N_e be in a constant ratio. The latter has already been assumed.

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7. For an example, see: Thompson, G. E. "An Investigation of the Local Employment Multiplier." Review of Economics and Statistics, Vol., 41, Feb., 1959, pp. 61-67.

Then $\beta_d N_d + \beta_e N_e = \beta_2 (N_d + N_e)$

from which $\beta_2 = (\beta_d N_d + \beta_e N_e)/(N_d + N_e)$

$= (\beta_d \epsilon + \beta_e) N_e / (\epsilon + 1) N_e$

$= (\beta_d \epsilon + \beta_e)/(\epsilon + 1)$ the required constant.

Export employment is retained in the rewritten structural relations by inserting $N_i = (\epsilon + 1) N_e$. Collecting the rewritten structural relations and adding disturbance terms produces

$$N = \ell_1 + \beta_1 Y + \mu_1$$

$$Y = \ell_2 + kI + \mu_2 \qquad N > 0$$

$$I = \ell + \beta_2 N_i + \mu_3 \qquad N_i > 0$$

$$N_i = (\epsilon + 1) N_e + \mu_4 \qquad N_e > 0$$

N_e is the sole exogenous variable in this recursive system. Progressively substituting for the explanatory variables N_i, I, and Y produces

$$N = (\ell_1 + \beta_1 \ell_2 + \beta_1 k \ell) + \beta_1 k \beta_2 (\epsilon + 1) N_e + (\beta_1 k \beta_2 \mu_4 + \beta_1 k \mu_3 + \beta_1 \mu_2 + \mu_1)$$

which can be denoted $N = \delta + \beta N_e + \mu$. This equation is referred to as one of the reduced forms in econometrics rather than a composite function as in mathematics.

The properties of a least squares estimator of the β parameter depend upon the probability distribution of the disturbance term μ, hence upon the probability distributions of the structural disturbance terms of which μ is a linear combination. In order for a least squares estimator of β to be a best linear unbiased estimator the following must prevail. 8/ The expectation of the structural disturbance terms must be zero for each time period, their variance-covariance matrix of contemporaneous disturbance

8. The material in this paragraph is based on: Anderson, R. L. and Bancroft, T. A. Statistical Theory in Research. McGraw-Hill Book Co., Inc. New York, N. Y. 1952. p. 60. and Johnson, J. J. Econometric Methods. McGraw-Hill Book Co., Inc. New York, N. Y. 1963. Chapters 7, 8 and 9.

terms must be constant through time, the covariance of non contemporaneous disturbances within each structural relation must be zero, and the covariance between noncontemporaneous disturbances of different structures must also be zero. The last two conditions impose zero autocorrelation and serial correlation of the structural disturbance terms, respectively. With these conditions, any regional aggregate disturbance term μ in the export employment multiplier formulation $N = \delta + \beta N_e + \mu$ will have zero expectation, constant variance through time, and zero auto-correlation, the conditions required for a least squares estimator of β to be a best linear unbiased estimator. It is sufficient for the least squares estimator to be unbiased if the expectations of the structural disturbance terms are equal to zero for every time period. If any of the other structural conditions required for least squares to be the best linear unbiased estimator break down, the sampling variance of β will ordinarily be increased.

An estimator of the export employment multiplier that does not assume the aggregate output-employment and consumption functions or the over-all N_d-N_e relation to be linear is the input-output analysis of W. W. Leontief. The price paid for the greater flexibility of an input-output estimator is that it cannot be implemented soley with regional total and export employment data. Information on sector input and output flows must also be gathered. An input-output model will be used later as it allows explicit analysis and estimation of indirect production and employment.

Export Employment Estimators

Estimates of export employment for implementing the three linear estimators of the export multiplier that have been discussed usually are based upon one or more of the following information sources: (1) a survey of regional firms to secure their total employment and export sales fraction

of total sales, (2) total employment figures by industry for the region being estimated and one or more comparison areas encompassing the region, and (3) judgement of persons making the export employment multiplier estimate as to the regional employment engaged in producing exports. Estimates utilizing the first two sources manipulate verifiable data in explicit formulas; assumptions sufficient for their accuracy will be developed.

In a survey determination of private sector export employment information as to total employment and the proportion of export sales is typically solicited from firms located in a region. An estimate of a firm's export employment is obtained by multiplying the export sales fraction times its total employment. 9/ In mathematical form this estimator, termed the survey estimator, is

$$\hat{N}e^* = \frac{E}{T} \cdot N$$

where E = export sales, sales to nonregional residents and establishments

T = total sales

N = total employment

N_e^* = direct export employment

The explicit assumption in this estimator is that export employment is in the same ratio to total employment as export sales are to total sales.

If a firm produces only one product, just one qualification is needed for the survey estimator to be correct; the firm must not have experienced a net addition to its finished inventory over the time period to which the employment and sales data relate. Otherwise, the sales fraction for estimating export employment will be too large. This condition, which will be assumed in the remainder of this section, is a subset of the multiplier requirement of zero regional inventory change discussed earlier.

9. Denver Planning office. Working Denver; an Economic Analysis. 1953. p.26.

If a firm produces two or more products, taking the case of two for an example, then the survey estimator becomes

$$\hat{N}_e^* = \frac{E}{T} N = \left(\frac{E_a + E_b}{T_a + T_b}\right) (N_a + N_b)$$

letting E_a = export sales of product A

E_b = export sales of product B

T_a = total sales of product A

T_b = total sales of product B

N_a = employment engaged in production of A

N_b = employment engaged in production of B

If the employment and export sales fraction data are not secured by product, the following must be true if the estimator is to be without error in the case of nonjoint products:

$$\left(\frac{E_a + E_b}{T_a + T_b}\right) (N_a + N_b) = \frac{E_a}{T_a} N_a + \frac{E_b}{T_b} N_b$$

To determine the conditions sufficient for the last equality to exist, the following relation will be used. If $E_a/T_a = E_b/T_b$, then $(E_a + E_b)/(T_a + T_b) = E_a/T_a = E_b/T_b$. So, if the export sales fractions are equal for all products produced by a survey respondent, the survey estimator of a firm's aggregate export employment will be correct. Alternatively it is sufficient for the accuracy of the estimator if the employment/total sales ratios are equal for all products. This follows because

$$\frac{E_a}{T_a} N_a + \frac{E_b}{T_b} N_b = \frac{N_a}{T_a} E_a + \frac{N_b}{T_b} E_b$$

A regional export employment total arrived at by use of the survey estimator will estimate direct private plus government user-cost-financed export employment. Without modification of the estimator, export employ-

ment would always be zero in construction and the remainder of government establishments. That employment of firms producing output which is incorporated into exports by regional purchasers will be omitted from the export employment estimate. Such employment has been referred to as indirect export employment, using input-output terminology.

An approximate survey estimator of indirect export employment has been suggested. 10/ Regional direct exporting firms would be asked what proportion of their input was purchased from regional firms in addition to the two previously mentioned questions. Indirect export employment of a firm attributable to sales to a regional exporter would be equal to the direct export sales fraction of the purchaser times the fraction of the seller's output going to him times the seller's employment. The approximate survey estimator of indirect export employment in mathematical form is

$$\hat{N}_{e_s} = N_s \left(\frac{L}{T_s}\right)\left(\frac{E_p}{T_p}\right)$$

letting N_{e_s} = indirect export employment of the selling firm attributable to exports of the purchasing firm

N_s = total employment of seller

L = sales of seller to a regional purchaser

T_s = total sales of seller

E_p = exports of purchaser

T_p = total sales of purchaser

An input-output estimator of regional employment attributable to direct and indirect exports is $\hat{N}_e = (b') \quad (I + A + A^2 + A^3 + \ldots)$ (e)

10. Leven, C. L. "An Appropriate Unit for Measuring the Urban Economic Base." Land Economics, Vol. 30, Nov., 1954, p. 370.

letting N_e = regional direct and indirect export employment

b = vector of linear and homogeneous output-employment coefficients

I = identity matrix

A = matrix of processing sector input coefficients with C, G, GPCF and E in final demand

e = vector of sector exports

Indirect export employment would be equal to $(b')\ (A + A^2 + A^3 + \ldots)\ (e)$ and one approximation would be $(b')\ (A)\ (e)$. To collate with the suggested survey estimator of indirect export employment, suppose two of the processing sectors were each composed of one firm and that one of those firms (s) sold output to the other firm (p). If none of that output were purchased for purposes of GPCF and if there were no inventory changes in the survey period, the truncated input-output estimator of indirect export employment in the s sector, $b_s A_{sp} e_p$, could be written $(N_s/T_s)\ (L/T_p)E_p$ using survey estimator notation. The latter expression is identical to the approximate survey estimator. However, if some of the seller's output were utilized for GPCF' by the purchaser, the p sector input coefficient from sector s (A_{sp}) would be overstated. This could cause the degree of approximation to vary from one exporter to the next, hence to vary among indirect exporters. Even if GPCF' equalled zero, this particular truncated estimator would not uniformly approximate indirect export employment because the power series $A + A^2 + \ldots$ does not converge uniformly for each sector. This is particularly serious for the truncation A and will be demonstrated in the subsequent empirical section. For both of these reasons, comparisons of indirect export employment among sectors and among regions estimated by the approximate survey estimator could be distorted.

The second type of data used in making estimates of regional export employment do not include information directly relating to export sales. Instead, a proxy variable is used, typically industry employment for one or more areas besides the region. There are many variations but a common one, stemming from the location quotient, estimates nonexport employment in industry j to be the same fraction of total employment in the region as the total industry j/total employment fraction in another area. The industry j export employment estimate is regional industry j employment minus the imputed nonexport employment if that difference is greater than zero. 11/ This estimator in mathematical form is:

$$\hat{N}_{e\#_j} = N_j - N(U_j/U) \quad \text{if} \geq 0$$

letting $N_{e\#_j}$ = export employment of a regional industry j (j = 1,2, .. n) restricted to be greater than or equal to zero

N_{o_j} = nonexport employment in regional industry j

N_j = total employment of regional industry j, assumed equal to $N_{e\#_j} + N_{o_j}$ if $N_{o_j} \leq N_j$

N = total employment of the region

U_j = industry j employment of a comparison economy including the region

U = total employment of the comparison economy including the region

Uj/U is a ratio estimator of the nonexport employment proportion in regional industry j. The estimate of the proportion will be denoted by $N_{o_j}\hat{/}N$. Substituting, the export estimator becomes $N_j - N(N_{o_j}\hat{/}N) = N_{e\#_j} + N_{o_j} - \hat{N}_{o_j} = \hat{N}_{e\#_j}$. The question of interest is how good is the estimator Uj/U of the ratio N_{o_j}/N?

11. See: Thompson, op. cit., p. 64.

A statistical property which is easily demonstrated for a ratio estimator when it is not implemented with proxy data is unbiasedness in the sampling limit. 12/ To illustrate, suppose N_{o_j}/N were estimated from a random sample of m firms out of a regional total of M by determining their total employment, their industry and - if in industry j - their nonexport employment.

Let $N_{o_j}/M = \bar{N}_{o_j}$ and $N/M = \bar{N}$

A ratio estimator of nonexport employment in industry j would be

$$N_{o_j}\hat{/}N = \sum_{k=1}^{m} N_{o_jk} \Big/ \sum_{k=1}^{m} N_k = m\hat{\bar{N}}_{o_j} / m\hat{\bar{N}} = \hat{\bar{N}}_{o_j} / \hat{\bar{N}}$$

If all M firms in a region were contacted, unbiasedness in the finite limit for this estimator is demonstrated by

$$N_{o_j}\hat{/}N = \sum_{k=1}^{M} N_{o_jk} \Big/ \sum_{k=1}^{M} N_k = M\bar{N}_{o_j} / M\bar{N} = \bar{N}_{o_j} / \bar{N} = N_{o_j}/N$$

Unbiasedness in the limit cannot be demonstrated for the U_j/U estimator of N_{o_j}/N, nor can any bias be measured, because it does not involve inductive inference from a sample of $N_{o_{jk}}/N_k$ ratios. In short, the implementation of an estimator with proxy data removes it from the class of estimators considered in the field of statistics. Any estimator implemented with proxy variables will hereafter be referred to as a proxy estimator.

How can a proxy estimator be evaluated other than by comparisons of estimates with the true values? A commonly employed method is to evaluate the a priori credibility of the assumptions and implied structural relations of a proxy estimator. 13/ This will be done for the proxy estimator of export employment.

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12. Cochran, W. G. Sampling Techniques. John Wiley and Sons, Inc. New York, N. Y. 1953. pp. 13 & 114.

13. For a simple example, see: Ferguson, C. E. "Comment." In: Elements of Regional Accounts, op. cit., pp. 47-49.

The explicit statement of the proxy estimator $\hat{N}_{o_j} = NU_j/U$ is that nonexport employment in regional industry j is the same fraction of total regional employment as the total industry j/total employment ratio exhibited by a comparison area. There are two possible interpretations of this - the actual regional ratio is indicated by the comparison area or the relative amount of employment the region would need to produce all of the industry j output used by nonexport demanders is indicated by the comparison area ratio. Since $\hat{N}_{oj}$ may be more than the total employment of industry j, the first possibility must be rejected. One implication of the second interpretation is that the comparison area has no exports or imports; such an economy is referred to as closed. If this is the case, then the total output and employment in the comparison area's industries is satisfying all the direct and indirect domestic demand, domestic demand consisting of consumption and domestic investment.

The following question is posed in order to start the development of explicit assumptions for the proxy estimator that would enable it to be generally correct, that is, the a priori conditions allowing the estimator to be applicable to any region. How much employment should be devoted to industry j in another closed economy of total employment size N? A simple and general rationale for the proxy estimator $\hat{N}_{o_j} = N U_j/U$ in this context is that the amount of industry j output necessary to satisfy linked domestic demand is in a constant proportion to that in the comparison area and that average labor productivities in the two areas, by industry and in aggregate, are also in a constant proportion. For the average labor productivity ratios to be constant, it is sufficient that the industry and aggregate output-employment functions be linear and homogeneous.

The following notation is adopted to indicate the aforementioned employment, output and demand relationships implicit in the proxy estimator of nonexport employment in a closed regional economy.

Let N = total employment in a region

U = total employment in a comparison area

j = industry subscript ($j = 1, 2, \ldots n$)

β = aggregate output-employment coefficient in the linear homogeneous function of the comparison area

λ = constant proportionality factor of output-employment coefficients in the region relative to the comparison area

$\tilde{O}_c$ = regional output needed to satisfy consumer (c) direct and indirect demand without any regional imports

$\tilde{O}_o$ = output in a region equal to $\tilde{O}_c + \tilde{O}_g + \tilde{O}_{gpcf}$

$\tilde{\Phi}_c$ and $\tilde{\Phi}_o$ = comparison area counterparts of regional $\tilde{O}_c$ and $\tilde{O}_o$

$\psi = \tilde{O}_{oj}/\tilde{\Phi}_{oj}$ $(j = 1, 2, \ldots n) = \sum_j \tilde{O}_{oj} / \sum_j \tilde{\Phi}_{oj} = \tilde{O}_o/\tilde{\Phi}_o$

The output-employment functions are written:

$$N = \lambda \beta \tilde{O}_o \qquad U = \beta \tilde{\Phi}_o$$

$$N_{o_j} = \lambda \beta_j \tilde{O}_{oj} \qquad U_j = \beta_j \tilde{\Phi}_{oj}$$

with $\sum_j \tilde{O}_{oj} = \tilde{O}_o$ and similarly for the other industry output and employment variables. There is no difficulty in maintaining the stability of the aggregate coefficient ratio λ because the relative distributions of industry output in the two areas have been assumed equal. The proxy estimator of nonexport employment in the closed region $\hat{N}_{o_j} = N\, U_j/U$ becomes, by substitution

$$\hat{N}_{oj} = \lambda\beta\, \tilde{O}_o \beta_j\, \tilde{\Phi}_{oj} / \beta\, \tilde{\Phi}_o$$

$$= \lambda\, \beta_j\, \tilde{\Phi}_{oj}\, \psi$$

and since $\sum_j N_{oj} = \sum_j (U_j/U)N = N$

It must be that $\lambda\, \psi = N/U$

therefore $1/\lambda = \psi\, U/N = (\tilde{O}_o/N)\ (\tilde{\Phi}_o/U)$

The last equality, required because the form and sense of the estimator constrains $\sum_j \tilde{N}_{o_j}$ to be equal to N, states ψ and λ to be dependent. The required relation is that the constant average ratios of labor productivities for the two areas $(1/\lambda)$ be equal to the linked domestic demand per employment ratios. One such relation is $\lambda = 1$ and $\psi = N/U$.

The use of the proxy estimator indicates the possibility that a region might have some employment devoted to export production. Hence, if total employment N is not that of a closed economy, it must be equal to it in order for N/U to represent an economic scalar of U_j. That is, the loss of regional employment producing direct and indirect exports must be equal to that which would be generated by the replacement, with regional production, of imports used to satisfy linked domestic demand. That equality will be assumed temporarily.

The preceding notation for the proxy estimator is now extended to include:

O = actual regional production
O_c = actual regional production going to consumer direct and indirect demand
$O_o = O_c + O_g + O_{gpcf}$
O_m = actual regional imports
O_{o_m} = imports directly and indirectly allocable to domestic demand
$\tilde{O}_{omj}$ = regional industry j production generated by regional replacement of all direct and indirect domestic demand imports
$\tilde{N}_{omj} = \lambda \cdot \beta_j \cdot \tilde{O}_{omj}$ = employment generated by $\tilde{O}_{omj}$
$N_{omj} = \lambda \beta_j O_{omj}$ = imputed regional employment lost to O_{omj}

The regional employment-output-demand relationships are now written:

$$N = \lambda \beta O$$
$$= \lambda \beta (O_c + O_g + O_{gpcf} + O_e)$$
$$= \lambda \beta (O_o + O_e)$$

assumed $= \lambda \beta \tilde{O}_o$

therefore $(\tilde{O}_o - O_o) = O_e$

so that $\lambda\beta (\tilde{O}_o - O_o) = \lambda\beta \cdot O_e = N_e$ is actual regional direct and indirect export employment.

Also $N_j = \lambda\beta_j O_j$

$= \lambda \beta_j (O_{oj} + O_{ej})$

$= \lambda \beta_j (\tilde{O}_{oj} - \tilde{O}_{om_j} + O_{ej})$

The proxy estimator of industry j export employment

$$\hat{N}_{e\#_j} = N_j - N U_j/U$$

becomes by substitutuion $= \lambda \beta_j (\tilde{O}_{oj} + O_{ej} - \tilde{O}_{omj} - \tilde{O}_{oj})$

$$= \lambda \beta_j (O_{ej} - \tilde{O}_{om_j})$$

$$= N_{e_j} - N_{\tilde{O}m_j} \quad \text{if} \geq 0$$

By this derivation, the proxy estimator of industry export employment is seen to be an estimate of the regional employment producing linked exports of industry j less the regional industry j employment that would result from the substitution of regional production for all imports consigned to linked domestic demand. This net amount, when greater than zero, hereafter will be referred to as net linked export employment.

An essential assumption of the proxy estimator is that actual linked export employment equals the regional employment that would be generated by the replacement of linked domestic demand imports. The analytical device employed to deduce conditions sufficient for this assumption to be generally correct will be the open nondynamic input-output model with linear homogeneous output-employment functions. Processing industries would exclude households (consumption), domestic gross public and private capital formation, and the direct demand components of multiplier exports. This would leave

domestic noncapital activities of regional government in the processing sector. 14/

Let b = vector of industry output-employment coefficients

A = matrix of regional input coefficients

B = matrix of regional input coefficients with regional production substituted for imports of processing sector output

a = vector of observed final domestic demand for regional processing industry output

e = vector of observed processing industry direct exports

m = vector of observed direct processing industry imports by the final domestic demand sector

o = vector sum of a + e

p = vector sum of a + m

It is required that $(b')\ (I - A)^{-1}\ (o) = (b')\ (I - B)^{-1}\ (p)$. An obvious way to achieve equality is to impose A = B and e = m. If A is to equal B there must not be any imports of processing industry output by the regional processing industries. The vector of direct exports (e) will equal that of domestic final demand imports (m) if direct exports of each regional industry j equals imports of industry j output. The most illuminating case to consider is A equal to B equal to the null matrix, that is, no inter or intra industry transactions as well as imports in the processing sector. This will rule out any indirect production in the region. It is then required that $(b')\ (I)\ (o) = (b')\ (I)\ (p)$ which reduces to $(b')\ (e) = (b')\ (m)$. Such an equality will always exist if the sum of regional processing industry exports equals the sum of imports of their output and if the elements of (b) are a constant ($\lambda\ \beta$).

14. As has been indicated by Perloff and Leven, op. cit., p. 198, a difficulty in including a component of non user-cost-financed regional government in this processing sector is the meaningfulness of the implied relation of such government expenditures to activity levels in a region. The above inclusion does permit a more meaningful statement of the required employment equality, however.

If the A matrix equals the B matrix equals the null matrix and all β_j equal β in the comparison area, it need not be a closed economy. The previous requirement that comparison area imports of industry j output and exports of industry j output both equal zero could be relaxed to having them equal.

If the regional A matrix equals the B equals the null matrix and all $\lambda\beta_j = \lambda\beta$, the proxy estimator of industry net export employment

$$\hat{N}_{e\#_j} = N_j - N\,U_j/U$$

can be written $= \lambda\,\beta\,(\tilde{O}_{o_j} - O_{om_j} + O_{e_j} - \tilde{O}_{o_j})$

$$= N_{e_j}^{*} - N_{om_j} \quad \text{if} \geq 0$$

The range on $\sum_j N_{e_j}^{\#} = \sum_j (N_{e_j}^{*} - N_{om_j})$

Is seen to be $0 \leq \sum_j N_{e\#_j} \leq \sum_j N_{e_j}^{*}$

The proxy estimator of industry net linked export employment in this case is then equal to direct export employme t minus the regional employment imputed to observed imports of industry j output by direct domestic demanders.

Linear homogeneous output-employment functions have been used thus far in the proxy estimator discussion. If linear nonhomogeneous i dustry and aggregate output-employment functions were postulated, the estimator would become trivial. This is because two constraints on the regression constants are necessary for the proxy estimator to be internally consistent. Denoting the regression constants by $\underline{N}j$ and $\underline{U}j$ for the region and comparison area respectively, the constraints are that $\underline{N}j/\underline{U}j = Nj/Uj$ for all j and that $\sum_j \underline{N}_j / \sum_j \underline{U}_j = N/U = \lambda\,\psi$. The first is necessary for Uj/U to correctly allocate regional employment to exports and other and the second is necessary for the total employment ratio to correctly indicate the ratio of total domestic demand in the two areas adjusted for labor productivity differences. To

generally realize both, however, the relative distribution of industry employment must be the same in the region and comparison area which means zero net export employment would be estimated for the region.

In order to generate the data needed for a valid test of the proxy estimator of regional net export employment the following are needed: an open nondynamic input-output model with an industry origin of all imports, a split of government expenditures in the region into direct export and other, a split of real capital investment into direct export and other, and linear homogeneous output-employment functions. The last feature would allow estimates of actual linked export employment and actual linked nonexport employment which would exhaust the total employment producing regional output. The breakdown of imports by industry would permit an estimate of the amount of regional employment attributable to supplying all regional direct domestic demand without any imports by the final domestic users or regional processing industries. The last employment estimate minus actual linked nonexport would represent domestic import replacing employment. The industry summation of linked export minus import replacing employment when greater than zero would represent more direct estimates of net linked export employment to compare with those produced by the proxy estimator.

A disturbing feature of the proxy estimator of net export employment is that as the number of industries into which a region's employment total is divided increases, the estimate - if it changes - can only increase. 15/ In general, net export employment will increase because industry disaggregation allows more opportunities for differences in the relative distribution of

15. Kansas City Federal Reserve Bank. "The Export-Local Employment Relationship in Metropolitan Areas." Monthly Review, March, 1960, p. 9 of technical appendix.

employment of a region and its comparison area. This type of monotonicity is not evident in the survey estimator as a firm is disaggregated by product nor in an input-output estimator as the number of subdivisions in the processing sector is increased.

A final point about testing the proxy estimator concerns the employment data used to implement it. Two types of industry employment data are available for many regions, the Census household series and an establishment report series. The household series relates to the industry attachment of employed residents of a region regardless of where employed. Establishment reports, with some exclusions, relate to jobs held in the region regardless of the residence of the job holder. Both have been used in implementing the proxy estimator. 16/ Loosely speaking, industry employment in the estimator denotes production in an area while the summation of the industry employment indicates domestic demand. Establishment employment would be expected to indicate the composition of industry production within an area more accurately than an employed resident series. The case for the superior total employment series is not so clear-cut but employed residents are probably a more constant indicator of population and hence of the level of domestic demand insofar as the latter two are correlated. From these considerations it is concluded that, for the proxy estimator, a region and its comparison area should be demarcated so that persons employed in the area and employed residents are as nearly identical as possible. The transition from persons employed in an area to jobs held in an area requires that the relative amounts of multiple job holding in the region and comparison area be similar. In the former case net linked export employment refers to employed persons and in the latter to jobs held.

16. See: Thompson, op. cit., p. 62.

Employment Multiplier Estimates

Some empirical content will be given to the preceding discussion of export employment and its linear and homogeneous multiplier by utilizing data from an input-output study of St. Louis for 1955. 17/ To do this, some modifications have to be made in the previous formulation of the export employment multiplier. First, the sectoring used in the input-output study grouped all gross private capital formation together without any export-nonexport classification. Of the federal, state and local government sectors, only local user-cost-financed activities were sectored as to export and non-export. This prevents adherence to the definitions of export and domestic multiplier investment that have been advanced. Because of this, two different sets of definitions which conform to the input-output sectoring will be used.

The first definition of exports consists of all sales of new production to nonresident persons and nonregional business and government establishments with domestic investment consisting of all gross private capital formation. These two definitions are referred to as the "trade" set. The purchases and employment of the local, state and federal government sectors will be largely excluded from the export employment multiplier estimates using these definitions. The second set of definitions includes the expenditures and employment of government establishments in the region by adding the state and federal sectors to the preceding exports and the local government sector to domestic investment. This set of definitions is referred to as "inclusive." The employment data used are from a St. Louis nonagricultural series and an agricultural census, they exclude the military, household domestics, unpaid family workers and the nonagricultural self-employed.

17. Hirsch, W. Z. "Interindustry Relations of a Metropolitan Area." Review of Economics and Statistics, Vol. 41, Nov., 1959, pp. 360-369. I am indebted to Professor Hirsch for allowing me access to the records of this study.

An estimate of the export employment multiplier using the linear homogeneous estimator and a counterpart of the survey estimator of export employment will be presented first. A survey estimate of export employment could not be constructed because output inventory changes within the producing firms could not be successfully disinterred from the study records. The alternative was to assume either that the regional inventory changes of sector output as recorded in the transactions matrix occurred entirely within the producing firms or entirely out of them. 18/ The latter alternative was judged to be the error minimizer with respect to the survey estimator of export employment. Utilizing this place of inventory change assumption, sector sales are equal to sector output totals, including inventory additions, minus inventory depletions calculated from the transactions matrix. This implementation of the survey estimator of export employment at the sector level yielded the following linear homogeneous estimate of the export employment multiplier for St. Louis.

(trade) $N/N_e^* = 644,406/208,838 \simeq 3.09$

(inclusive) $N/N_e^* = 724,600/245,266 \simeq 2.95$

The inclusive estimate of export employment would be equal to that produced using the survey estimator of export employment at the firm level if the place of inventory change assumption is correct, if all firms in a sector had equal export sales proportions or equal employment/total sales ratios, and if state and federal employment in the region were considered export.

18. For the inventory recording procedure, see: Evans, W. D. and Hoffenberg, Marvin. "The Interindustry Relations Study for 1947." Review of Economics and Statistics, Vol. 34, May, 1952, p. 108.

Linear homogeneous export employment multiplier estimates incorporating the equilibrium requirement of no regional inventory changes can be computed. The problem is to know what a consistent set of industry outputs would have been in the survey period if the inventory changes that were present had not occurred. Such equilibrium production can be estimated with the input-output estimator at hand because multiplier investment, as well as exports, has been defined in terms of St. Louis input-output sectors. Industry outputs were estimated corresponding to the observed multiplier investment output sector distribution at the time of the St. Louis study. These industry outputs and their export final demand components were converted into employment using linear homogeneous output-employment functions. An aggregate export employment multiplier was then computed as the sum of the processing sector equilibrium employment, plus all government employment in the region for the inclusive set, divided by total export employment. These aggregate estimates correspond to weighting the sector export employment multipliers by their relative amount of regional export employment in the survey period. The export employment multiplier estimates are:

(trade) $N/N_{e*} = 556,275/208,838 \simeq 2.66$

(inclusive) $N/N_{e*} = 696,283/245,266 \simeq 2.84$

These estimates, which are smaller than the preceding ones, reflect the exclusion of nonexport employment attributable to the net increase in regional inventories during the survey period. The range on the total/export employment sector ratios for the trade definitions was from 1.03 for a manufacturing category to infinity for the construction industry. For the inclusive definitions, the range was from 1.03 for the same manufacturing sector to 41 for another nonmanufacturing category. About two-thirds of the ratios of each set fell within the values of one to three.

Subjected to the conditions of the previous paragraph and the constancy of sector exports as a fraction of the prescribed sector final demand, a regional nondynamic input-output estimator will always produce an aggregate estimate of the export employment multiplier equal to that given by the linear homogeneous estimator implemented with a survey estimate of inclusive export employment for the same time period if (1) regional inventory changes did not occur in the survey period, (2) all firms in a processing sector have equal export/total sales or employment/total sales ratios and (3) state and federal government are considered export.

Utilizing the assumptions of the linear homogeneous estimator, the export employment multiplier becomes

$$\frac{\Delta N}{\Delta N_e} = \frac{N}{N_e} = \frac{N}{Y} \frac{Y}{D+E} \left(\frac{D}{N_d}\frac{Nd}{N_e} + \frac{E}{N_e}\right)$$

All of these quantities can be estimated with input-output. A difficulty with the spending measure estimates will be that state and federal transfer payments must be included in exports instead of domestic investment in the inclusive set because of the input-output sectoring. To have total employment equal to employment attributable to exports, domestic investment and consumption, these component employment figures must include indirect as well as direct employment. All production can be attributed to consumption plus multiplier investment by excluding these sectors from the processing sector, hence from the corresponding inverse output matrix. The use of linear homogeneous sector output-employment functions then attributes all regional employment to the linked production values so that $N = N_c + N_d + N_e$. Performing this on the St. Louis data produces

$$\text{(trade)}\quad \frac{N}{Ne} = \frac{556,275}{6,818,772}\,\frac{6,818,772}{4,856,299}\left(\frac{860,839}{74,659}\,\frac{74,659}{283,133}\,\frac{3,995,460}{283,133}\right) \approx 1.96$$

$$\text{(inclusive)}\quad \frac{N}{N_e} = \frac{696,283}{8,540,538}\,\frac{8,540,538}{5,873,952}\left(\frac{1,069,252}{100,334}\,\frac{100,334}{326,252}\,\frac{4,804,700}{326,252}\right) \approx 2.13$$

These smaller multiplier values reflect the interfirm linkage in St. Louis causing indirect export employment to be over one-third of direct. The range on the total/linked export employment sector ratios for the trade definitions was from 1.01 to nine and from 1.01 to ten for the inclusive. Slightly more than two-thirds of the ratios of each set fell within a range of one to two.

The truncated input-output estimator of indirect export employment (b') (A) (e) discussed in connection with the approximate survey estimator was implemented, using the St. Louis data. The resulting estimates were

(trade) $\widehat{N/N_e} = 556,275/264,027 \simeq 2.11$

(inclusive) $\widehat{N/N_e} = 696,283/305,655 \simeq 2.28$

The ratio of the truncated estimate of indirect export employment to the exhaustive estimate for the trade and inclusive definitions of exports was .74 and .75 respectively. The ratio of the truncated to the exhaustive estimates for each of the twenty-odd processing industries were also computed to test for sector uniformity of the approximations. The sector proportions varied from 65 to 92 per cent with approximately half of them falling outside the range 70 to 80 for both the trade and inclusive sets. This is interpreted as a rejection of a uniform approximation (covergence) hypothesis. At best, the uniformity of the approximate survey indirect export employment estimator is that apparent in the truncation of the power series approximation to the desired inverse matrix of output coefficients. The reason for the sector disparity of the estimator can be appreciated by considering the sum of the geometric series $r + r^2 + r^3 + \ldots$ with zero less than r, r less than 1. Letting r equal .4 and .6, for example, it is observed that the ratio of r to the sum of its corresponding series is .6 and .4, respectively.

The investment and export income multipliers have been suggested as estimators of the linked investment and export employment multipliers. 19/ Corresponding to the linear homogeneous estimator the income multipliers are Y/I and Y/E. The I and E components of Y have been measured in terms of producers prices. They could be measured in other monetary terms, value-added for example. Accordingly appropriate value-added aggregates were approximated by computing the sum of the household and government row inputs corresponding to the same linked production values used to compute linked export and domestic investment employment. The multiplier estimates are:

Investment	N/N_I	Y_{v-a}/I_{v-a}	Y/I
(trade)	1.55	1.48	1.40
(inclusive)	1.63	1.50	1.45
Export	N/N_e	Y_{v-a}/E_{v-a}	Y/E
(trade)	1.96	1.88	1.71
(inclusive)	2.13	1.93	1.78

The export employment multipliers are greater than the investment because the domestic investment which is assumed to accompany exports converts into investment employment, but not export. All the dollar ratios are slightly lower than their corresponding employment ratios. The estimates are interpreted, however, as not rejecting the hypothesis that total employment, linked investment and export employment are proportional to their corresponding linked value-added aggregates. The revenue ratios, which are smaller than the value-added ratios, are understated because the heavily consumer oriented sales of the large trade sector were recorded in the transactions matrix in terms of trade margins rather than prices. The inclusive total/export revenue

19. See. Hansen, A. H. A Guide to Keynes. McGraw-Hill Book Co., Inc. New York, N. Y. 1953. p. 87 and Fox, Karl. "The Study of Interactions Between Agriculture and the Nonfarm Economy: Local, Regional and National." Journal of Farm Economics, Vol. 44, No. 1, Feb., 1962, p. 23.

ratio is also understated because of the previously mentioned inclusion of government transfer payments in exports. A hypothesis of proportionality of total and linked investment and export employment to the corresponding direct producers revenue is not considered to be tested by these estimates because of the indicated nonconformities of the data. The estimates are regarded as suggesting that such a hypothesis has promise.

Summary and Conclusions

This has been an examination of estimators of the regional aggregate export employment multiplier corresponding to the coefficients in three simple linear regressions of total employment on export employment. The structural relations underlying the reduced forms, which are the total employment-export employment-regression equations, have been demonstrated. The regression coefficient estimators correspond to input-output employment multipliers which can be condensed to one number because the sector distribution of final demand and its export and domestic components are assumed to change only by a scalar and the sector output-employment functions are assumed to be linear. The assumption of a constancy of the change in endogenous domestic investment to that in exogenous exports constitutes a comparative statics aggregative domestic capital formation sub-model. As has been indicated by others, the export employment multiplier model to which the three estimators conform assumes that there is no feedback of regional imports upon regional exports. 20/ A more inclusive definition of exports than the usual trade composition has been advanced to strengthen its designation as the sole exogenous variable determining changes in total employment. Any rigorous empirical implementation of that definition would almost certainly involve more work than an aggregate linear estimate of the multiplier would be worth as a quantitative predictor. This is because the enumerated assumptions which allow the input-output sector employment multipliers to be condensed to one number, coupled with the consumption and indirect production assumptions of the input-output estimator, are judged to be excessively rigid. A linear and homogeneous export employment multiplier estimate would have somewhat greater value as an extrapolator than as a predictor.

20. Tiebout, C. M. "Exports and Regional Economic Growth." The Journal of Political Economy, Vol. 64, April, 1956, p. 162.

Estimates of the linear and homogeneous investment and export linked employment multipliers using input-output data for St. Louis are judged to not reject the hypothesis of equality with their corresponding income multiplier estimates. The range on the various total to linked and unlinked export employment ratios presented for that region was from 2.0 to 3.1. This interval would probably cover the majority of aggregate export employment multiplier estimates that have been published for regions in the United States.

There are several conceptual types of total employment for a region and even more types of export employment. Employment in a region is superior to employed residents for purposes of identifying export employment because exports have been defined in terms of the disposition of regional output. The identification of exporters and their export employment is judged to yield more information about a region than an aggregate linear export employment multiplier estimate. Export employment may be that directly connected with export production or that directly and indirectly connected. The survey estimator of direct export employment, on a physical establishment basis to reduce vertical integration effects, is recommended. The approximate survey estimator of indirect export employment is not recommended because it has been indicated to be susceptible to serious capital formation and sectoral rate of convergence errors. The input-output estimator of indirect export employment is recommended, of course. The proxy estimator of net linked export employment remains empirically untested, but the realization of its assumptions for an urban region appears dubious. Pending the outcome of such tests to indicate sensitivity, its use is not recommended.